IO'S INTERNAL HEAT DISSIPATION

A Simplified Mathematical Model

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Abstract

Jupiter's moon Io owes its intense volcanic activity to tidal forces acting to distort its crust. This distortion of Io's crust represents an increase in energy. Io releases this energy through volcanic heat. A mathematical model of this system represents Jupiter as a large point mass and Io as two masses connected by a damped spring. This nonlinear system has no analytical solution and obtaining a numerical solution involves the use of intense computations. The computing facilities available make a simplified model of the Io-Jupiter system more sensible.

Introduction

We have attempted to model the tidal interactions of Io with Jupiter in order to explain Io's abnormally intense internal heating and volcanic activity. Discovered in 1610 by Simon Marius and Galileo Galilei, Io, with a mass of $8.9\text{x}10^{22}$ kilograms and an equatorial radius of 1,815 kilometers, is the third largest of Jupiter's known satellites. It is the innermost of the Galilean moons, located a mean distance of about 422,000 kilometers from Jupiter. Having an eccentricity of 0.004, Io's orbit around Jupiter is completed in approximately 1.8 Earth days.

Io's molten silicate rock surface shows numerous volcanic calderas, many of which lie along active hot spots. 61 active centers of volcanism have been identified since the discovery of Io's volcanism in 1979 by Voyager 1 (Lopes-Gautier 243-244). This fact indicates that Io has an active interior, with eruptions of sulfur or sulfur dioxide. Hot spots indicate the heating of Io's interior and its consequent heat transfer (Lopes-Gautier 249).

Interactions between Io and its fellow Galilean moons, Europa and Ganymede, with Jupiter have been defined as the source of Io's active lifestyle. A 4:2:1 resonance exists among these three moons respectively, meaning that for each orbit of Ganymede, Europa orbits Jupiter two times, while for each orbit of Europa, Io makes two orbits around Jupiter. The influence of the Galilean moons forces Io to maintain an eccentric orbit around Jupiter. Without this influence, Io's orbit would move toward circularity due to the dissipation of tidal energy in Io's interior (Ojankangas and Sevenson 341-342).

Io is directionally locked in its orbit, meaning that the same side always faces Jupiter, a quality exhibited by the Moon toward Earth. But, the gravitational forces from Europa, Ganymede, and Jupiter add a slight "wobble" to this directional locking so that a straight line connecting the two masses does not always extend to the center of Jupiter (Arnett 1998). The gravitational force on the near side of Io therefore changes slightly over time, giving rise to large tidal forces. The gravitational attractions of these bodies cause a bending and stretching of Io in an elastic fashion (Segatz 194). This "tug-of-war" leads to up to 100 meters of tidal bulging on Io's surface (Hamilton 1999). These tidal interactions heat Io's interior, which contributes the energy of its volcanic activity.

The Model

Our goal is to develop a simple model of Io's motion around Jupiter that produces two of the three major effects of tidal dissipation and directional locking. The third major effect, orbital circularization, should not result from this model as we expect this to result from small perturbations in Jupiter's surface due to Io's gravitational pull. This model does not take any tidal flow on the surface of Jupiter into account.

We will treat the tidal motion as 2 masses connected by a damped spring; the damping will model the tidal dissipation. The 2 masses represent halves of Io being distorted by Jupiter and nearby moons. The compression and expansion of the spring represent this stress from tidal forces. The damping corresponds to the tidal dissipation. The rest length of this spring equals the average diameter of Io, as the surface of the planet has the least potential energy if its shape is spherical. The spring constant relates to the strength of Io's tidal flows. Io's surface moves much more than most planets under tidal forces. For comparison, the Earth's crust moves about 0.4 mm in 12 and 24-hour periods due to tidal forces from the moon as opposed to 100 meters for Io.

The damping coefficient relates to the rate at which Io releases energy in the form of heat. We expect significant power loss to occur over time periods much greater than one orbital period, as significant tectonic plate movements occur over periods of thousands of years. The rate of power loss due to tidal dissipation can give us a value for this damping coefficient.

Equations of Motion

The first step is to derive the equations of motion for the system. Io's gravitational force on Jupiter can be safely ignored due to Jupiter's immense size if we are only looking for tidal dissipation and orbital locking. Using the gravitational potential energy, the potential energy of the spring, and the kinetic energy, the Euler-Lagrange Equation gives the force in each coordinate. The damping term can then be added to the appropriate coordinates without loss of generality. In order to apply the damping term appropriately, polar coordinates provide an intuitive view of the physical system. The center of mass position of Io, R, and the angular displacement of the center of mass, Φ, locate Io in its orbit. The orientation of the mass-spring system relative to the center of mass includes Θ, the angle the system makes with the orbital radius, R. r_1 and r_2 then describe the distance from Io's center of mass to the two masses. The mathematics allows for a substitution of r for r_1 - r_2 giving a total of four coordinates for a complete description of the system (see Fig. 1).

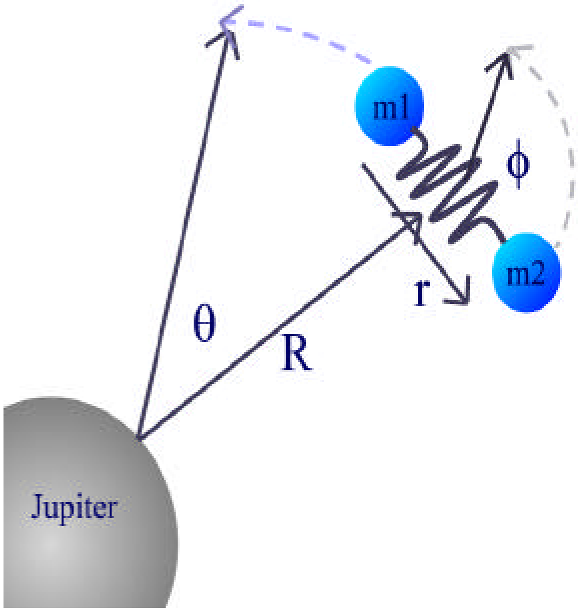

Figure 1

The potential energies of the two masses of Io are:

$$U=\frac{-G\cdot m_1\cdot M}{R}\cdot\left(1+\frac{r}{R}\cdot\cos(\Theta)+\frac{1}{4}\cdot\frac{r^2}{R^2}\right)^{\frac{-1}{2}}+\frac{-G\cdot m_2\cdot M}{R}\cdot\left(1+\frac{r}{R}\cdot\cos(\Theta)+\frac{1}{4}\cdot\frac{r^2}{R^2}\right)^{\frac{-1}{2}}$$

The total potential energy of Io divides into that of one mass at location R + r plus that of the other mass at location R – r. In this system, r << R allowing a second order Taylor Series expansion to be used as an approximation to the above equation.

$$U=\frac{-G\cdot m_{Io}\cdot M}{R}\cdot\left[1+\frac{1}{2}\cdot\frac{r^2}{R^2}\cdot\left(3\cdot\cos(\Theta)^2-1\right)\right]$$

The undamped equations of motion follow from the Euler-Lagrange relation. The damping force only affects r, the relative position of the two masses. This represents the dissipation of tidal energy.

Solutions and Observed Effects

Directional locking implies a stable equilibrium of rotational motion. An accurate model should demonstrate directional locking. Again, this means that Io would rotate with a period equal to its revolution period, implying that the same side of Io always faces Jupiter. Our model has the ability to exhibit directional locking. If the two sides of Io are given a large enough initial angular velocity about their center of mass, one side will be closer to Jupiter at one time and the other side will be closer at a later time. If, over the time period of numerical integration, one side is consistently closer to Jupiter, then Io is directionally locked.

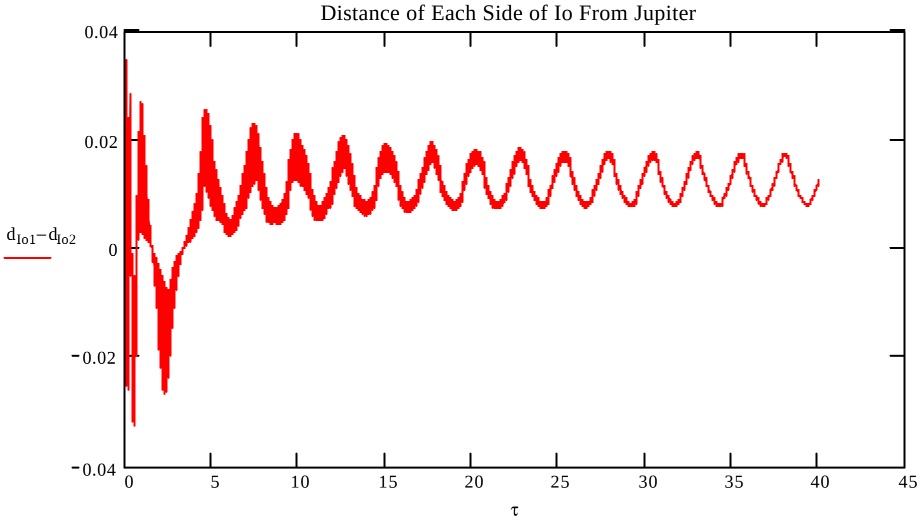

Figure 2

Here, we see that the first side of Io remains farther from Jupiter than the second side. We can also see the "wobble" of the directional locking discussed previously.

Another stable equilibrium of orbital motion involves circularization. As mentioned previously, we expect orbital circularization to result from a model that takes tidal flows on the surface of Jupiter into account. A look at the following effective potential versus radial distance graph (fig. 3) demonstrates the stable equilibrium in a 2-body gravitational interaction. L=angular momentum, and μ = reduced mass of the system.

$$U_{eff}(r_{Io}) := \frac{L^2}{2\cdot\mu\cdot r_{Io}^2} - \frac{G\cdot M\cdot m}{r_{Io}}$$

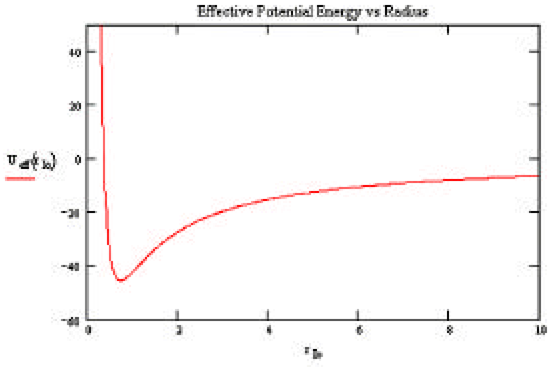

Figure 3

The stable equilibrium corresponds to the minimum on the graph, the orbit where the radius is constant over time. This represents a circular orbit. If a planet starts in an elliptical orbit, one can imagine a horizontal line extending across the graph at an effective potential energy between –45 and 0. As time progresses, energy inevitably leaves the system, causing the horizontal line to gradually move down until it reaches the minimum – a circular orbit. The model did not exhibit circularization, as expected.

Another observed effect of this model involves resonance. Resonance is “a state in which one orbiting object is subject to periodic gravitational perturbations by another” (Arnett 1999). Resonance occurs when the stretch of the spring due to its oscillations coincides with the stretch due to the gravity of Jupiter. Io revolves around Jupiter with a certain frequency. At the same time, it is also being deformed (stretched and compressed) by the tidal forces with a certain frequency. Resonance occurs when the frequency of the oscillation of the moon itself coincides with the time that it takes Io to revolve around Jupiter. This means that the distance between the two halves of Io (r) would be greater during resonance, because the combined effect of both the pull of Jupiter and the outward force of the stretched spring are acting on Io. In a mass-spring model exhibiting nearly linear oscillations, one could derive an approximate value for a resonance frequency. The following graph (fig. 4) illustrates Io’s oscillations moving in and out of a certain resonance frequency over time.

damping coefficient	spring constant	initial angular velocity
$\gamma = 0.01$	$k_{Io} = 50.133$	$\Theta'_0 = 0$

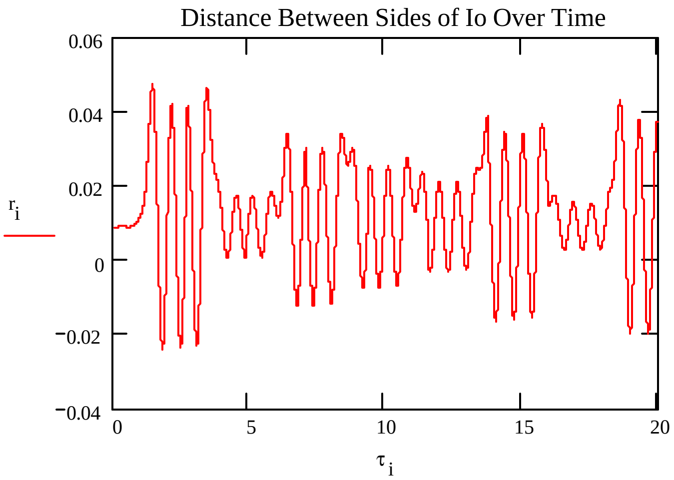

Figure 4

The model also unexpectedly displayed a precession of Io’s orbit (fig. 5):

R_0, Φ_0="initial position of Io" R_N, Φ_N="final position of Io"

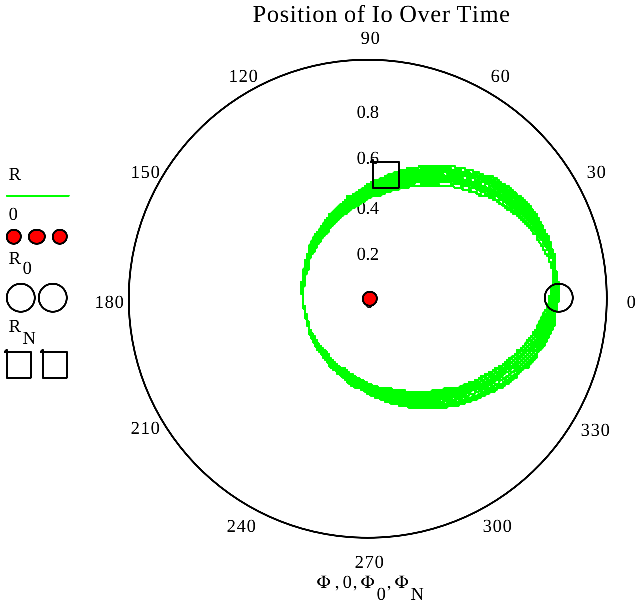

Figure 5

Although the Earth's precession about the Sun relates to its oblateness (Baierlein 179), we cannot explain Io's orbital precession in this instance. Perhaps additional moons would keep this in a stable orbit. We save this for future research.

Conclusion

Perhaps, in the future, time will allow us to design a model in which Jupiter's surface distorts slightly as well. This would involve a model in which both Io and Jupiter are modeled as double-mass, damped spring systems. On the other hand, our simplified model displays several interesting properties that should be expected of an accurate description of such a system, such as resonance, directional locking, and orbital precession.

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