AUTOCORRELATION

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Abstract

A titanium-doped sapphire laser produces pulses approximately 100 fs in length. To verify this, we used a process known as autocorrelation to find the duration of one laser pulse, by measuring the laser in relation to itself. First, the laser beam was split and sent on different, but equal paths. The two beams were focused through a lens into a potassium diphosphate crystal at an angle of 5° to the normal resulting in an output with $\lambda \approx 400\,\text{nm}$. The output beam that we were interested in had intensity equal to the product of the input intensities. There were two more output intensities that were equal to the doubling of each individual input intensity. It was found that the optic axis of the crystal should be 44.4° from the propagating beams. Results showed the appearance of the two doubled beams, but not of the product intensity.

Introduction

While many lasers produce a continuous beam of light, some lasers, including titanium-doped sapphire lasers, produce pulses. The pulses from a titanium-doped sapphire laser are separated by only about 12 ps (1 ps = 10^{-12} s) and last about 100 fs (1 fs = 10^{-15} s). This time scale is extremely short. Even the best oscilloscopes cannot measure times shorter than picoseconds. The best way to measure the pulse length of this laser is to measure it in relation to itself. This process is called autocorrelation, using the laser's own pulses to measure the duration of those pulses.

Although we know, approximately, what the pulse length of the laser is, we still want to have a way to determine this value. In order to make sure that the laser is working properly, and producing pulses that are indeed 100 fs long, one would have to find a way to measure this time length. Also, if one wanted to change the pulse length for any reason, resulting in a longer or shorter pulse, one would have to use autocorrelation to determine this duration.

Theory and Apparatus

<u>Theory</u>

Autocorrelation allows us to measure the length of a laser pulse by relating it to itself. In this experiment, we split the laser pulses, sending each half on separate paths, allowing them to intersect in a nonlinear crystal. The resultant intensity is proportional to the product of the two initial beam intensities. We then measure this product as a function of the time delay that we imposed between the two pulses. This measurement allows us to calculate the pulse length of the laser.

Figure 1a shows the intensity of two pulses, as functions of time, with one of those pulses delayed by a time, $\tau = 100$ fs. The product of the pulses as a function of time is illustrated in Figure 1b. This product is proportional to the intensity of the output beam. Figures 2a and 2b show what happens when the time delay is decreased to 50 fs. Notice that as the time delay decreases, the intensity of the product of two pulses increases. Figure 3a and 3b show that when $\tau = 0$, so that no time delay introduced, the output intensity is larger still. We see that the product is a maximum when the time delay is zero and it falls off sharply when the time delay becomes comparable to the pulse length. In the next section, we will quantify this observation.

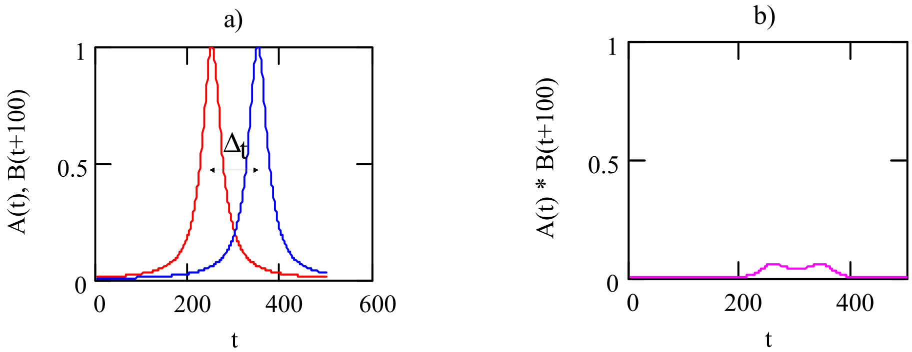

Figure 1: Fig. 1a shows the two pulses separated by a time delay of 100 fs. Fig 1b shows the product of the two pulses.

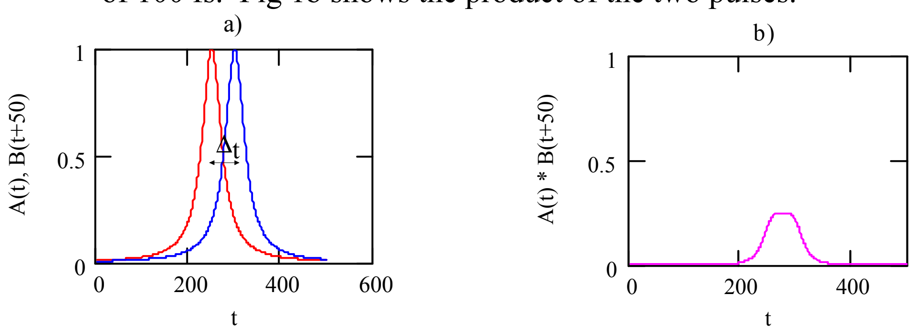

Figure 2: Fig. 2a shows the two pulses separated by a time delay of 50 fs. Fig 2b shows the product of the two pulses.

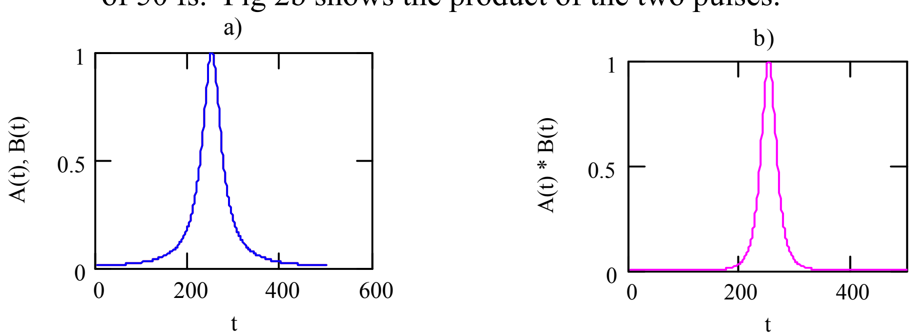

Figure 3: Fig. 3a shows the two pulses separated by a time delay of 0 fs. Fig 3b shows the product of the two pulses.

The size of the product can be quantified by the area under the product (its integral). Figure 4 shows the integral of the product of the two pulses as a function of time delay. Notice that as time delay decreases, the integral of the product increases. Therefore, when time delay equals zero, we find a maximum value for the integral of the product of the two pulses. This integral of the product as a function of time delay is called the autocorrelation of the beams. The autocorrelation is related to the width of the original pulses.

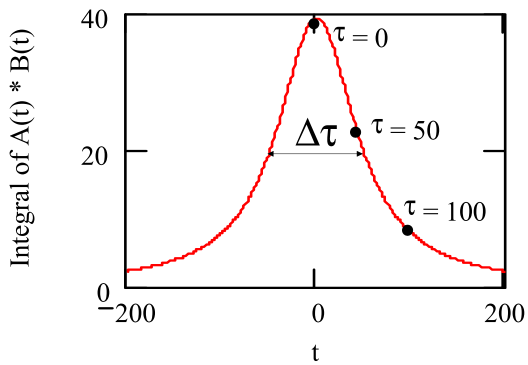

Figure 4: Plot of the integral of the product of time delayed pulses vs. time delay, τ.

We measure the width at the half maximum intensity point. If the original pulse shape is Gaussian, then this autocorrelation will be $\sqrt{2}$ times wider that the original pulse.

Experimental Apparatus

The apparatus used to build this autocorrelator is shown in Figure 5. The pulses are produced by a titanium-doped sapphire laser that emits a near infrared beam with wavelength 800 nm (1 nm = 10^{-9} m). We used a beam splitter to create two beams that can experience different time delays. The two paths that the beams traveled had to be nearly equal in length to ensure that the pulses in each beam met at the same place at the same time. If the paths differed by more than the pulse length, then the laser pulses would not intersect in the nonlinear crystal. Since the duration of each pulse was 100 fs, and since the length of the pulse equals the product of the pulse duration and the speed of light, c, then each pulse should have a length of 30 μm (1 μm = 10^{-6} m). It was not possible to measure the path lengths to an accuracy of 30 μm, but we could get as close as possible before making fine adjustments with the translation stage. We see in Figure 5 that 2a is the length of Path 1 and 2b+c is the length of Path 2, so we adjusted the apparatus until 2a = 2b +c within the accuracy of our measurements.

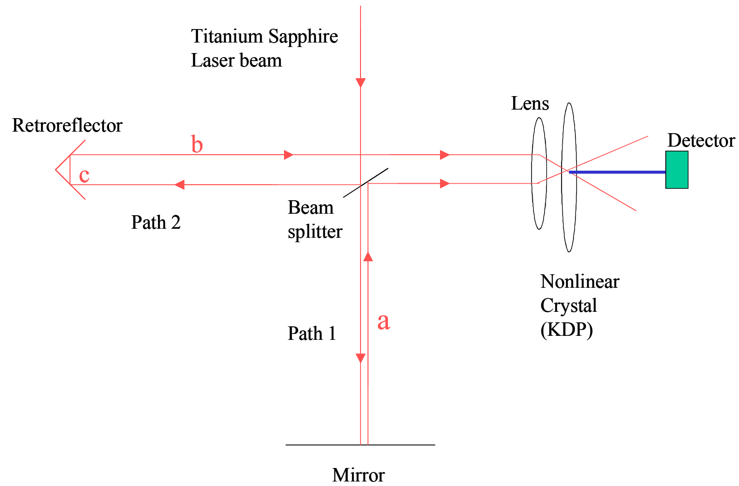

Figure 5: Experimental apparatus for autocorrelation.

The retroreflector was mounted on a translation stage to further adjust the distance traveled in Path 2, while the mirror along Path 1 remained stationary. After reflection from the mirror, retroreflector, and beam splitter, the beams traveled parallel to each other towards a planar convex lens. The focus of the lens was found by observing the point where the two beams converged. At this point of convergence, a potassium diphosphate (KDP) crystal was placed. A KDP crystal is non-linear; therefore, an incoming beam with a given input wavelength may produce a different output wavelength. We needed a non-linear crystal because it produced a signal that was proportional to the product of the two input intensities. The input beams were in the near infrared region of the spectrum, with wavelength of approximately 800 nm. When combined inside the KDP crystal, the result was a wavelength of approximately 400 nm; this is the wavelength range that corresponds to blue light. This resulting blue wavelength, measured by a detector, should show us if the two beams have in fact converged at the same place, at the same time. The detector used in this experiment was slow compared to the length of the pulse, since there are no detectors that can resolve the pulse length. Since we only needed to know the integral of the product, a slow detector was sufficient. It could tell us the average values of the power in the blue beam, which was proportional to the integral. Measuring the power of the blue beam as a function of time delay yields a signal that is proportional to the autocorrelation of the pulses.

We wanted to measure the autocorrelation as a function of time delay, and varying the length of Path 2 by adjusting the translation stage attached to the retroreflector varies the time delay. If d is the distance the translation stage moves, 2d is the change in path length, and c is the speed of light, then time delay, τ, is $\frac{2d}{c}$. In addition, there was another output worth mentioning. As a result of the doubling of the frequencies of each individual input pulse, two additional blue beams were created. However, we were only interested in the output resulting from the sum of the frequencies of both input pulses, not the doubling of each input pulse. These beams were distinguished because they propagated in different directions (see Figure 5).

Phase Matching

One extra factor that was considered was phase matching. Phase matching means adjusting the orientation of the KDP crystal so that the index of refraction for the blue light and the red light is the same. We know that when traveling through materials, the speed of light is reduced: $v = \frac{c}{n}$, where n is the index of refraction and c is the speed of light in vacuum. The index, n, generally decreases with increasing wavelengths, making it so that red light propagates faster than blue light. As discussed in the previous section, we used a nonlinear crystal to generate blue light from two input red beams. The blue light was generated gradually as the beam propagated through the crystal. For example, as illustrated in Fig 6, at some point, x, a portion of red light was changed into blue light, and the rest remained the same. And at another point, x+L, another portion of red light was converted to blue light. Little by little, red light was converted to blue light.[1]

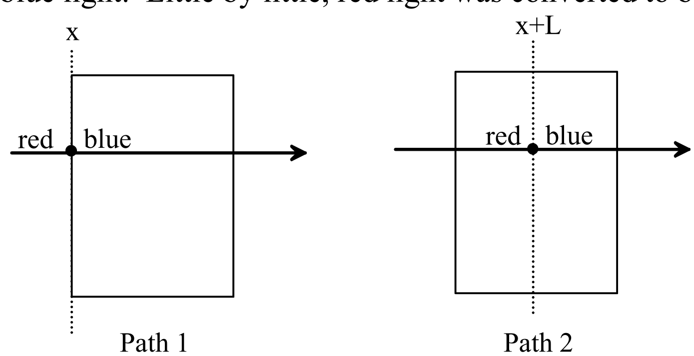

Figure 6: Red light being converted to blue light in small increments as if traverses through the KDP crystal.

Index matching was essential for the blue light generated at different points to be in phase. If the two blue beams are in phase, they will add up, increasing the overall intensity of the output beam. Likewise, if all of the blue beams are in phase the intensity of the output beam will increase. The ideal situation would have been for blue light generated at each point in the crystal to be perfectly in phase. However, this was difficult to achieve. We'd like to find the limit for how out of phase each beam can be before no light is emitted. If the light created at each point, x, was exactly out of phase with light emitted at x+L, then they would interfere destructively. If L is half the thickness of the crystal, then each bit will have a canceling partner and no blue light will be emitted. If we let v_b equal the velocity of blue light, v_r equal the velocity of red light, n_b equal the index of refraction of the blue light, and n_r equal the index of refraction of red light, then $v_b = \frac{c}{n_b}$ and $v_r = \frac{c}{n_r}$, by the definition of the index of refraction.

Therefore, if t_1 equals the time it takes for the blue light in path 1 to pass through the crystal and d equal the thickness of the crystal, then

$$t_1 = \frac{d}{v_b} = \frac{n_b d}{c}.$$

And if t_2 equals the time it takes for the blue light *and* the red light in path 2 to pass through the crystal, then

$$t_2 = \frac{n_r d}{2c} + \frac{n_b d}{2c}.$$

We can solve for the condition at which the relative time delay would be half the period, resulting in destructive interference.[2] Having a relative time delay of half a period would result in each pulse being half a wavelength out of phase with each other, adding to zero, thereby, canceling each other out. Limiting the time delay to less than the destructive interference condition, we require $\Delta t < \frac{T}{2}$, where T is the period. The requirement $\Delta t < \frac{T}{2}$ lends to the following conditions on the indices:

$$\begin{aligned}
\frac{T}{2} &> t_2 - t_1 \\
&> \frac{n_r d}{2c} + \frac{n_b d}{2c} - \frac{n_b d}{c} \\
&> \frac{n_r d}{2c} - \frac{n_b d}{2c} \\
cT &> (n_r - n_b)d \\
\lambda_b &> (n_r - n_b)d \\
\frac{\lambda_b}{d} &> n_r - n_b
\end{aligned}$$

Thinking of this relationship in terms of destructive interference allowed us to solve for the conditions for which the pulses could combine in a way that will not destructively interfere.

This relationship tells us that the difference between the index of refraction for red light and the index of refraction for blue light must be less than the wavelength of blue light divided by the thickness of the crystal. Using $d = 5\,\text{mm}$ (the thickness of our crystal), and $\lambda_b = 400\,\text{nm}$, we found that the difference between the two indices of refraction must be less than 8×10^{-5}. Therefore, to avoid having destructive interference in the output, we must make sure that the difference between the two indices is no greater than 8×10^{-5}. Thus, $n_r - n_b$ must be as small as possible. Achieving the ideal condition of $n_r = n_b$ is called phase matching. In most materials this would be impossible, since

$n_r < n_b$. It is possible only because of the special properties of KDP, the doubling crystal.

Most materials are isotropic, meaning the index of refraction is the same for every direction. However, some materials are anisotropic, which means that the index of refraction changes according to the direction of polarization (The direction of polarization of light is defined to be the direction of its electric field.). One type of anisotropic material is called uniaxial, which is what our crystal was. Uniaxial materials have two separate indices of refraction, depending on the orientation of the material.

Following the explanation of the index ellipsoid as illustrated by Yariv[3], one can make a graph of the index ellipsoid equation:

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1 .$$

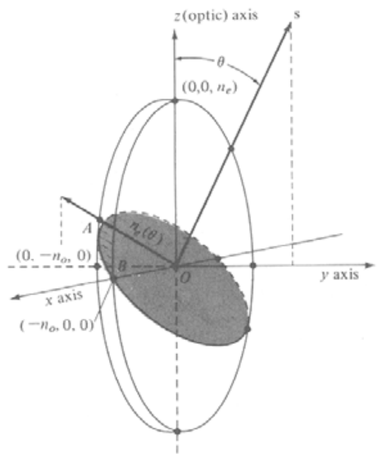

Figure 7: Construction for finding indices of refraction as light propagates in direction **s** through uniaxial crystal.[4]

Figure 7 illustrates the direction of propagation along vector **s** at an angle θ from the z-axis. The shaded portion of the ellipsoid, forming an ellipse, was the result of the intersection of the plane normal to the direction of propagation. The two polarization directions are along the ellipse axes, which are perpendicular to the direction of propagation. They are shown as segments of OA and OB, with $n_o = |OB|$ and $n_e = |OA|$.

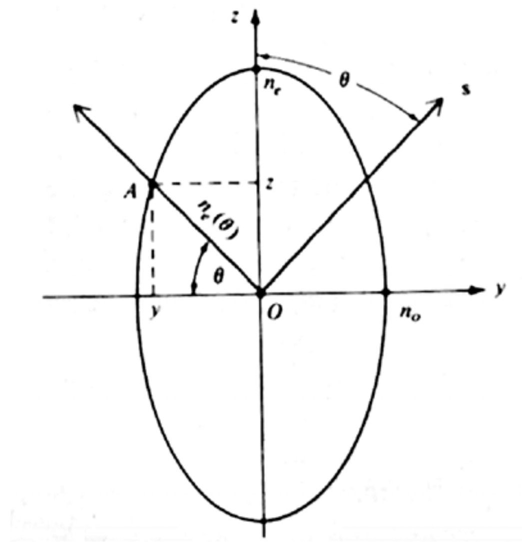

Figure 8: Intersection of the index ellipsoid with the y-z plane. [5]

Looking at the y-z plane of the index ellipsoid, as seen in Figure 8, we are able to see how the extraordinary index of refraction can vary depending on the angle of the propagating wave. The ordinary index of refraction remained fixed. For this experiment, the goal was to find a way for the index of refraction for blue light to equal that of red light. As stated earlier, red light was our input and it was polarized along the ordinary axis. Because the index of the ordinary axis was fixed, we had to adjust the extraordinary axis for blue light so that $n_{\text{red}} = n_{\text{blue}}$, thus achieving phase matching.

Since we know that the x and y direction are equivalent, we were free to define x = 0 wherever we want in the x-y plane. This allowed the direction of the propagation of the wave to lie in the y-z plane. Setting x = 0 in the equation for the index ellipsoid yields:

$$\frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 .$$

Using the Pythagorean theorem, we have the equation

$$n_e^2(\theta) = y^2 + z^2 ,$$

and using the relations

$$\sin\theta = \frac{z}{n_e(\theta)} \text{ and } \cos\theta = \frac{y}{n_e(\theta)} ,$$

we can solve for θ using the ordinary and extraordinary indices values for both red and blue light (see Table 1), we can solve for θ. Inserting the value of $n_{o,800\,nm}$ for n_e (because we want $n_{e,\,400\text{ nm}}$ to equal $n_{o,\,800\text{ nm}}$), we find that θ = 44.4°. This angle was the angle at which the laser beam should have entered with respect to the optic axis. It was at this angle that we were able to have the conditions for phase matching.

Table 1: Ordinary and Extraordinary indices at 400 nm and 800 nm[6]

Wavelength (nm)	Index at Ordinary Axis (n_o)	Index at Extraordinary Axis (n_o)
400 nm	$n_o = 1.524481$	$n_e = 1.480244$
800 nm	$n_o = 1.501924$	$n_e = 1.463708$

Results and Discussion

The beams were sent toward the lens in a direction parallel to each other and perpendicular to the lens. The focal length of the lens was 5 cm. The KDP crystal was cut such that the beams would make the desired angle of 44.4° if they enter at angle of 5° to the normal direction. As seen in Figure 9, with a 5 cm focal length, the angle will be 5° if the beams are separated by a distance 0.9 cm on the lens. At this focal point, the KDP crystal was placed with its optic axis perpendicular to the direction of polarization of the input beams (see Figure 9).

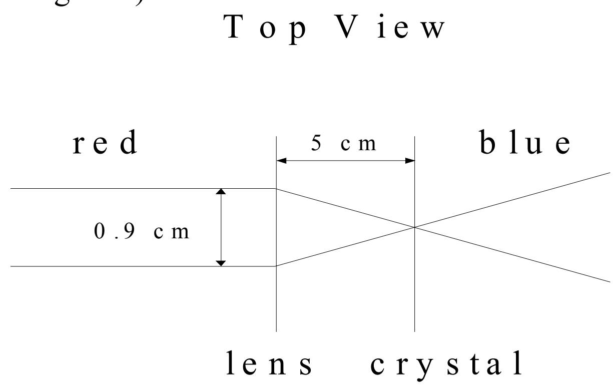

Figure 9: Top view of beams propagating towards lens and focusing to a point at the crystal. This was the original arrangement of the beams.

Because we know that phase matching should occur at θ = 44.4°, resulting in blue light output, we adjusted the orientation of the crystal. A resultant blue light should mean that the angle between the optic axis and the direction of propagation is at 44.4°. However, I observed that the two blue beams that resulted from the doubling of the frequency of the individual input beams appeared one at a time. This meant that the optic axis was 44.4° away from each propagated beam at different orientations. This finding showed that the optic axis was oriented in the same plane as the beams. Therefore, this setup had to be altered. Adjustments in the experimental setup called for the beams to travel parallel to each other aligned vertically, not horizontally (Figure 10). Since each input beam is vertically polarized and should be polarized along the ordinary axis, the optic axis must lie on the horizontal plane. If the two beams converge along the vertical plane, they will both make the same angle to the optic axis.

This adjustment was made by rotating the retroreflector in a way that changed the offset of the reflected beam. The beam was redirected so that it propagated toward the lens, vertically parallel to the other beam.

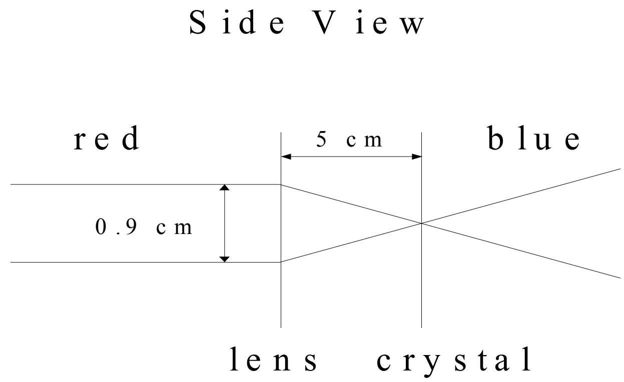

Figure 10: Side view of beams propagating towards lens and focusing to a point at the crystal. This is the new arrangement of the input beams. The retroreflected beam was redirected in a way that would allow both input beams to propagate vertically parallel to each other. The optic axis lies in the horizontal plane at an angle 44.4° to the two beams.

So far, we've been able to find the conditions for which the doubled beams occur in our output. But we're interested in the autocorrelated beam, so we must now use this information to find the angle between the autocorrelated beam and the optic axis. By aligning the crystal to maximize the power of the doubled beams, we have experimentally aligned the optic axis 44.4° from the input and doubled beams. Now we need to see what the angle would be between the optic axis and the autocorrelated beam, and how far off this angle is from the optimal angle of 44.4°. Using a three dimensional diagram, we can visualize this angle (see Figure 11).

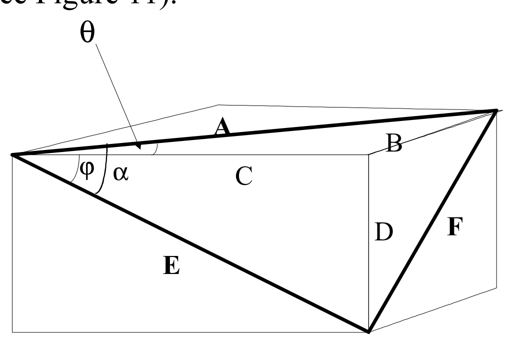

Figure 11: Three-dimensional view of one doubled beam and autocorrelated beam in relation to the optic axis.

If we let Line A represent the optic axis and Line E be one of the doubled beams, then the angle formed is α. This angle α is experimentally set to the optimal angle of 44.4° and the angle of the doubled beam to the normal is $\varphi = 5°$. We are interested in finding the angle between the line C (the autocorrelated beam) and the optic axis. This angle will be called θ. We can use geometry to find θ. The lengths of each leg of triangle AEF can be expressed in terms of C and the angles θ and φ:

$$A = \frac{C}{\cos\theta} \qquad E = \frac{C}{\cos\varphi} \qquad F = \sqrt{C^2 \tan^2 + C^2 \tan^2 \varphi}$$

These lengths can be inserted into the law of cosines:

$$F^2 = A^2 + E^2 - 2AE\cos\alpha .$$

We find (after some algebra) that $\cos\theta\cos\varphi = \cos\alpha$. Knowing that α= 44.4° and $\varphi = 5°$, we find that $\theta = 44.2°$. This means the autocorrelator beam would be 0.2° off from the optimal orientation. We experimented with the doubled beams to see how far off from the optimal orientation they could still be seen. We found that the crystal could be displaced form the optimal orientation by several degrees without losing the doubled signals. Therefore, we assume that the phase matching condition for the autocorrelation beam is met.

In the experiment, the crystal was oriented so that both doubled beams were clearly visible. Beam intensities were maximized by fine-tuning the orientation of the mounted crystal. A detector, connected to an oscilloscope, was placed in the assumed position of the product beam. This intensity was to be measured as a function of the change in path length. It is in this change in path length that creates the time delay necessary for autocorrelation. However, the detector did not detect a product beam.

Conclusion

The lack of the product beam could be due to a number of things. Maybe the product intensity was indeed there, but very faint. The detector may not have been as sensitive as it needed to be. But since has worked before, more likely, the two input beams may not have focused at the same exact point. Remember, it is necessary for each beam to focus at the same point so that photons from the two beams can combine to form and autocorrelator beam. However, the fact that we saw doubled beams implies that phase matching was achieved. If the doubled beams are phase matched, we have shown that the autocorrelated beam is also. Perhaps future experiments should focus on ensuring that the two beams are focused on the same point. This could be accomplished by using a pinhole and aligning each beam to the pinhole. In addition, due to its probable faintness, increasing the sensitivity of the detector may help in sensing the autocorrelated beam.

[1] M.E. Parks, Private Communication (2002).
[2] Ibid.
[3] A. Yariv, *Optical Electronics 3rd ed.*, 12-15, CBS College Publishing (1985).
[4] Ibid.
[5] Ibid.
[6] A. Yariv, *Quantum Electronics 2nd ed.*, 424, John Wiley & Sons, Inc. (1975).

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